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Over-Rotating Black Holes, Gödel Holography and the Hypertube

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Abstract

We demonstrate how a five dimensional Gödel universe appears as the core of resolved two-charge and three-charge over-rotating BMPV black holes. A smeared generalized supertube acts as a domain wall and removes regions of closed timelike curves by cutting off both the inside and outside solution before causality violations appear, effectively allowing the Gödel universe and the over-rotating black hole to solve each other's causality problems. This mechanism suggests a novel form of holography between the compact Gödel region and the diverse vacua and excitations of the bound state of a finite number of D0 and D4-branes with fundamental strings.

1 Introduction and Summary

One of the most productive approaches in theoretical physics is the development of toy models to extract key physical insight into the behavior of related problems which are either technically intractable or very complex. One current theoretical problem of great import is the understanding of de Sitter spacetimes in the context of a consistent theory of quantum gravity such as string theory; such spacetimes are widely believed to describe the very early universe. Of particular interest would be a good picture of how holography applies to de Sitter space.

A possible toy model for understanding de Sitter holography was proposed by Boyda etal. in [1], where they pointed out that a supersymmetric Gödel universe solution [2] shared several intriguing properties with de Sitter space. In particular, they showed the existence of observer-dependent finite-size preferred holographic screens (in the nomenclature of [3]) associated with any localized observer in this spacetime – a qualitative feature reminiscent of the holographic properties of cosmological horizons of de Sitter space. In contrast to the puzzling case of the de Sitter universe, however, the classical Gödel solutions of string theory typically exhibit a large number of Killing spinors, raising the hope of using supersymmetry to control the behavior of these solutions in full string theory.

Unfortunately, while the Gödel solution is endowed with interesting holographic properties, it also has closed time-like curves through every point. Although this spacetime has a globally well-defined timelike Killing vector (thereby formally allowing supersymmetry), there is no Cauchy surface, and no manifestly consistent inner product for quantum fields, making it difficult to even formulate any preliminary conjecture for a holographic correspondence between the degrees of the freedom in the bulk spacetime and the hypothetical holographically dual degrees of freedom on any of the locally defined compact screens.

Another observation in [1], however, perhaps offers a clue for further progress. If one selects a single observer in the Gödel universe, the corresponding preferred screen carves out a tubular region of the causality-violating classical Gödel solution which in itself is causally safe. This led the authors of [1] to conjecture that holography could play a crucial role in resolving the causality problems in some solutions of string theory, effectively suggesting the possibility of "holographic chronology protection." The intriguing structure of holographic screens in the Gödel universe has been determined in [1] by purely kinematic arguments, using Bousso's "phenomenological" definition of preferred screens; however, no insight into the microscopic theory of such hypothetical screens was offered in [1]. Thus, a natural question emerges: Are there *dynamical* objects in string theory that would play the role of the holographic screens in the Gödel universe? If this question has a positive answer, such

objects would be codimension-one domain walls in spacetime, and their degrees of freedom should be connected to those of the bulk Gödel universe via holographic duality.

In [4], Drukker etal. demonstrated that a supertube [5, 6] had just the right sort of worldvolume to bound a tubular region in a three-dimensional Gödel universe with seven compact directions. Unfortunately, this solution has a few flaws for our purposes: it is not clear why the supertube should have a radius small enough for the region within to exclude CTC's, and the asymptotic metric suffers from logarithmic divergences.

In order to use Gödel as a toy model for gaining an understanding of de Sitter holography, we would like a domain wall sitting at some radius smaller than that at which closed timelike curves appear, and such that the outside solution is asymptotically flat (or has some other "good" asymptotic behavior at infinity, for example, asymptotes to AdS space). Asymptotic flatness seems useful, since it allows one, for example, to introduce black holes inside Gödel [7, 8, 9, 10, 11] while simultaneously maintaining a reference for notions of mass and temperature (or even the conventional definition of a horizon), otherwise thorny problems in the absence of nice asymptotic behavior at infinity (see [9]).

In this paper, we will take as motivation the results of [4], and use smeared supertubes to bound the five-dimensional Gödel solution of [2]. We find an outside solution which takes the form of a reduction of the extremal rotating BMPV black hole [12, 13, 14, 15] obtained by setting one of the three charges of the BMPV solution to zero.[1] We will also consider as our outside solution the full three-charge rotating BMPV black hole, which requires adding a dust of wrapped D4-branes to the domain wall supertube. The supertube/D4-brane system is U-dual to the well studied D1/D5/KK system; we will use this to examine the microstates of our solutions. Finally, we will propose a novel form of holography relating the Gödel bulk degrees of freedom to those of the domain wall. We will not provide the full explicit construction, but rather set out the outline of a future program outside the scope of this paper.

Our presentation will be organized as follows. In Section 2 we will provide the relevant supergravity solutions for the five-dimensional Gödel universe and its possible "completions" outside the domain wall. The Israel matching conditions provide the crucial input for Section 3, where we detail the microscopic properties of the domain walls for the two-charge and three-charge black holes. We will not be considering the most general domain walls for sourcing these black holes, but rather look at those directly relevant to the Gödel solution.

[1] It is tempting to be somewhat cavalier about the terminology, and keep referring to the two-charge solutions or the over-rotating three-charge solutions as "black holes," despite the fact that they carry no macroscopic Bekenstein-Hawking entropy and actually represent naked singularities if taken seriously as classical solutions of low-energy supergravity.

In Section 4 we will give near-horizon (or near-tube) limits, while Section 5 will provide some deeper analysis with comments on the smearing of the supertubes, the relations to the D1/D5/KK CFT and the near-tube limit. We will close with an outline of our ideas for Gödel holography and some broader general remarks.

Just before this article was posted, another paper appeared on a similar subject [16].

2 Creating a Divide: The Inside and Outside SUGRA Solutions

The starting point for our paper will be a general supertube IIA supergravity solution as described by Emparan, Mateos and Townsend in [6]:

$$
\begin{aligned}
ds^2_{IIA} &= -U^{-1}V^{-1/2}(dt-A)^2 + U^{-1}V^{1/2}dy^2 + V^{1/2}\sum_{i=1}^{8}(dx^i)^2, \\
B_2 &= -U^{-1}(dt-A)\wedge dy + dt\wedge dy, \qquad e^{\Phi} = U^{-1/2}V^{3/4}, \\
C_1 &= -V^{-1}(dt-A)+dt, \qquad C_3 = -U^{-1}dt\wedge dy\wedge A.
\end{aligned}
\tag{2.1}
$$

It turns out that both the Gödel solution and the corresponding class of (the two-charge reduction of) BMPV black holes can both be rewritten in this form (or, in the three-charge case, a slight generalization thereof). Here y is a coordinate on S^1 of radius R_9, and the x^i parametrize the flat $\mathbf{R}^4\times T^4$, with the volume of T^4 denoted by V_4. Also, instead of the flat coordinates x^i, $i=1,\dots 4$ on the $\mathbf{R}^4$, we will frequently use spherical coordinates (r,ϕ,ψ,θ), with the conventional Euler angles $\phi\in[0,4\pi)$, $\psi\in[0,2\pi)$ and $\theta\in[0,\pi)$. In such spherical coordinates, the functions U and V as well as the one-form A (in the basis of invariant Euler one-forms on S^3) depend on r only. The ansatz will satisfy the equations of motion if U, V and A are all harmonic on $\mathbf{R}^4$.

The supertube spacetimes are actually special cases of the IIA reduced M-theory solutions of Gauntlett, Myers and Townsend [17] with M5-brane charges set to zero. The absence of M5-branes means that the solutions generically preserve eight supersymmetries. The inside solution and the first outside solution will take the supertube metric form above. The inside, a five-dimensional Gödel universe, is a very symmetric supertube spacetime, with twenty supersymmetries. On the other hand, our second outside solution will require a slightly more general framework, with one non-zero M5-brane charge in the M-theory lift and thus only four supersymmetries.

Before we go on to describe these solutions, we would like to spotlight their symmetries. We limit our attention to solutions that respect the $U(1)_L\times SU(2)_R$ subgroup of the maximal possible rotation symmetry group $SO(4)$ of four space dimensions. This condition restricts the angular momentum (or vorticity) tensor to be self-dual. Our rotation symmetry group

acts transitively on 3-spheres, and therefore suggests a natural class of adapted coordinate systems, in which the orbits of $U(1)_L \times SU(2)_R$ are squashed 3-spheres of constant radial coordinate r. In such coordinates, the angular momentum (or vorticity) of the solutions is captured by the off-diagonal components of the metric that mix dt with one of the Euler forms on the squashed S^3. We will not discuss the addition of $U(1)_R$ charge until we get to Section 4.

2.1 The Inside: Gödel Universe

Using the Euler one-forms σ_i, $i = 1, \dots 3$ on S^3, we can write the string metric for the basic $\mathcal{N} = 1$ minimal five-dimensional supergravity Gödel solution in a manifestly $U(1)_L \times SU(2)_R$ invariant form, as:

$$ds_5^2 = -(dt^2 + \beta \frac{r^2}{2} \sigma_3)^2 + dr^2 + \frac{r^2}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \tag{2.2}$$

with a gauge field

$$\mathcal{A} = -\frac{\sqrt{3}}{2} A = \frac{\sqrt{3}}{4} \beta\, r^2 \sigma_3. \tag{2.3}$$

This solution has closed time-like curves. For example, if we write σ_3 as $d\phi + \cos\theta\, d\psi$, then when $g_{\phi\phi} = \frac{1}{4} r^2(1 - \beta^2 r^2)$ becomes negative, i.e. when $\beta r > 1$, the orbits of ∂_ϕ will become time-like. On the other hand, it was observed in [1] that the portion of spacetime bounded by a wall at any radius $r \leq 1/\beta$ is free of causality violations, since any closed timelike curve in the full spacetime would have to leave the bounded region before returning to its original point. A phenomenological approach to holography further indicated that the natural holographic screens – as defined by an observer localized at the origin of the spherical coordinates in the Gödel geometry – should be surfaces of some constant value $r = r_H$, strictly smaller than $1/\beta$.[2] Thus, placing a domain wall at some value of $r < 1/\beta$ will have the beneficial effect of curing the causality problems of the classical solution, in a way suggested by the holographic principle.

We can lift this 5D solution to a 10-dimensional IIA one as follows [2]:

$$\begin{aligned} ds_{10}^2 &= ds_5^2 + dy^2 + \sum_{i=5}^{8} (dx^i)^2 \\ B_2 &= A \wedge dy, \qquad C_3 = A \wedge (x^5 \wedge x^6 - x^7 \wedge x^8) \end{aligned} \tag{2.4}$$

[2]The effective (super)gravity approach of [1] suggested the value of $r_H = \sqrt{3}/2\beta$ for the location of the preferred screen. One should expect that in full string theory the value of r_H at the location of the "optimal" holographic screen can in principle undergo a finite shift, with the precise value determined from some yet-to-be-developed microscopic theory of holography for the Gödel solution.

To get to the supertube form of this metric we T-dualize along the x^5x^6 plane, and then we dualize the corresponding 5-form potential to a 3-form potential (A is self-dual in the 1234 subspace). This leaves the NS-NS fields unchanged, while the RR fields now become

$$C_1 = A, \qquad C_3 = -dt \wedge dy \wedge A. \tag{2.5}$$

Clearly, this Type IIA spacetime belongs to the class described by our main ansatz (2.1), with the choice of functions $U = 1$, $V = 1$, $A = -\frac{1}{2}r^2\sigma_3$. In this form this solution is a dimensional reduction of the M-theory plane-wave with 24 supercharges, as emphasized in [18].

We will use this lift to M-theory to briefly review the supersymmetries. The metric in eleven dimensions can be written as

$$ds_{11}^2 = 2e^+e^- + e^ye^y + e^ie^i \tag{2.6}$$

with 1-forms

$$e^+ = dx^+, \quad e^- = -(dx^- - A), \quad e^y = dy, \quad e^i = dx^i. \tag{2.7}$$

The M-theory four-form is simply

$$F_4 = -dx^+ \wedge dy \wedge dA. \tag{2.8}$$

In this basis, it is convenient to define three projectors

$$\Gamma_-, \qquad \frac{1}{2}\, dA|_{ij}\, \Gamma^i\Gamma^j \equiv dA \cdot \Gamma, \qquad \text{and} \qquad \frac{1}{2}\,(1 - \Gamma_y). \tag{2.9}$$

We will label a basis of 32 spinors in blocks of four, $\varepsilon^{\pm\pm\pm}$, with a "+" if they are projected out by the respective projector, and with a "−" otherwise. The Killing spinor equations require that spinors either be projected out by Γ_- or $dA \cdot \Gamma$, so this M-theory solution has 24 Killing spinors. The spinors which satisfy $\Gamma_-\,\varepsilon = 0$, $\varepsilon^{+\pm\pm}$, give sixteen standard [19] supersymmetries while the remaining eight, $\varepsilon^{-+\pm}$, are supernumerary.

Of the twenty-four spinors, sixteen are constant:

$$\varepsilon^{+++}, \qquad \varepsilon^{++-}, \qquad \varepsilon^{+-+}, \qquad \text{and} \qquad \varepsilon^{-+-}. \tag{2.10}$$

The spinors ε^{-++} have non-trivial x^i-dependence, while the spinors ε^{+--} depend on x^+. The latter dependence means that we reduce the M-theory plane-wave to our Gödel solution, only 20 supercharges are preserved.

2.2 The Outside Part I: The Two-Charge Reduction of the BMPV Black Hole

The 10D IIA supergravity solution for a rotating two-charge reduction of the BMPV black hole [12, 13], with D0 and F1 charges Q_0 and Q_s respectively, also takes the form of a supertube metric with eight supersymmetries:

$$\begin{aligned} ds^2 &= -\tilde{U}^{-1}\tilde{V}^{-1/2}(d\tilde{t} + \frac{J}{4\tilde{r}^2}\,\sigma_3)^2 + \tilde{U}^{-1}\tilde{V}^{1/2}d\tilde{y}^2 + \tilde{V}^{1/2}\sum_{i=1}^{8}(d\tilde{x}^i)^2, \\ B_2 &= -\tilde{U}^{-1}(d\tilde{t} + \frac{J}{4\tilde{r}^2}\,\sigma_3)\wedge d\tilde{y} + d\tilde{t}\wedge d\tilde{y}, \qquad e^{\Phi} = \tilde{g}_s\,\tilde{U}^{-1/2}\tilde{V}^{3/4}, \\ \tilde{g}_s C_1 &= -\tilde{V}^{-1}(d\tilde{t} + \frac{J}{4\tilde{r}^2}\,\sigma_3) + d\tilde{t}, \qquad \tilde{g}_s C_3 = \tilde{U}^{-1}d\tilde{t}\wedge d\tilde{y}\wedge \frac{J}{4\tilde{r}^2}\,\sigma_3 \end{aligned} \tag{2.11}$$

with

$$\tilde{U}(\tilde{r}) = 1 + \frac{Q_s}{\tilde{r}^2}, \qquad \tilde{V}(\tilde{r}) = 1 + \frac{Q_0}{\tilde{r}^2}. \tag{2.12}$$

These fields are written in the string frame; we find it convenient to exhibit $\tilde{g}_s$-dependence in the RR-fields. The eight supersymmetries are those of all supertube spacetimes, with $\tilde{r}$-dependent Killing spinors proportional to ε^{+-+} and ε^{+++} [6].

This solution has two pathologies. First, the dilaton-gradient and metric-curvature diverge at $\tilde{r} = 0$. Second, like the Gödel solution, this metric also develops CTC's: orbits of the vector field ∂_ϕ with radii smaller than a critical value $\tilde{r}_c$ satisfying

$$4\,\tilde{r}_c^2\,(\tilde{r}_c^2 + Q_0^2)\,(\tilde{r}_c^2 + Q_s^2) = J^2 \tag{2.13}$$

provide a clear example. In fact, if we restrict ourselves to the region *outside* this critical radius, there are no CTC's. We now see that an appropriate match of the inside solution (Gödel) with the outside solution (BMPV) could remove all closed time-like curves (as well as the pathological throat region).

We are interested in matching this outside solution to our inside Gödel solution at a fixed radius $\tilde{R}$. At that point, the harmonic functions take values

$$\chi_s \equiv \tilde{U}(\tilde{R}) = 1 + \frac{Q_s}{\tilde{R}^2}, \qquad \chi_0 \equiv \tilde{V}(\tilde{R}) = 1 + \frac{Q_0}{\tilde{R}^2}. \tag{2.14}$$

In order for the dilaton and the metric to remain real, we also require $\chi_s, \chi_0 \geq 1$, i.e. we will only consider putting our domain wall at positive $\tilde{R}^2$. This restriction is of course implied by the fact that $\tilde{r}^2 = 0$ is a singularity.

When we cut off the outside solution at any finite value $\tilde{R}$ of the $\tilde{r}$ coordinate and try to match it with the Gödel solution, the $\tilde{t}, \tilde{r}, \tilde{x}^i, \tilde{y}$ coordinate system does not represent a smooth continuation of the coordinate system that we used in the previous subsection to

describe the Gödel solution. For example, the inside solution always has $g_{rr} = 1$ while for the outside solution the value of $g_{\tilde{r}\tilde{r}}$ at the fixed cut-off radius is $\sqrt{\chi_0}$. This discrepancy is an artifact of using a coordinate system which is not smoothly extended across the domain wall. The correct way to define what is a smooth coordinate system is to demand the equality of *all* the metric components at the wall, not just of the metric induced on the wall. Two natural smooth coordinate systems suggest themselves:

(1) One can extend the $(\tilde{t}, \tilde{r}, \tilde{x}^i, \tilde{y})$ coordinate system from the outside to the region inside the domain wall. This requires specific constant rescalings of the original Gödel coordinates. The advantage of this coordinate system is that the metric now asymptotes to the canonical Minkowski metric at infinity, at the cost of introducing inconvenient rescaling constants in the Gödel region at the core of the solution.

(2) Alternatively, one can keep the original Gödel coordinates (t, r, x^i, y) on the inside, and extend them to the outside region by constant rescalings of the original outside coordinates. We find that this coordinate system is better suited for the description of the region near the domain wall, at the price of losing the nice asymptotic property of the metric components at infinity. We will refer to this coordinate system as the "near-wall" or "near-tube" coordinate system.

The rescaling between the two coordinate systems takes the following form:

$$\tilde{t} = \chi_s^{1/2} \chi_0^{1/4}\, t, \qquad \tilde{y} = \chi_s^{1/2} \chi_0^{-1/4}\, y, \qquad \tilde{x}^i = \chi_0^{-1/4}\, x^i. \tag{2.15}$$

In order to match at the domain wall, both geometries must induce the same metric on the wall itself. To match the volumes of T^4, S^1 and S^3, as well as the squashing of the S^3, we require that

$$\tilde{V}_4 = \chi_0^{-1} V_4, \quad \tilde{R}_9 = \chi_s^{1/2} \chi_0^{-1/4} R_9, \quad \tilde{R} = \chi_0^{-1/4} R, \quad J = \chi_s^{1/2} \chi_0^{-1/4}\, 2\beta R^4. \tag{2.16}$$

The string coupling and the six-dimensional Newton constant at the domain wall are

$$\tilde{g}_s = g_s \chi_s^{1/2} \chi_0^{-3/4}, \qquad \tilde{G}_6 = G_6 \chi_s \chi_0^{-1/2} \tag{2.17}$$

If we place our domain wall at a radius $\tilde{R} > 0$ such that we cut-off our space time before the dilaton gets singular, the χ's take values in the interval $[1, \infty)$. It is convenient to define new parameters

$$\gamma_s = 1 - \chi_s^{-1}, \qquad \gamma_0 = 1 - \chi_0^{-1} \tag{2.18}$$

which take values in the interval $[0, 1)$. In terms of our new parameters γ and the new coordinates, we can now write the outside BMPV solution using the ansatz (2.1) with the

following choice of U, V and A:

$$U = 1 + \gamma_s(\frac{R^2}{r^2} - 1), \qquad V = 1 + \gamma_0(\frac{R^2}{r^2} - 1), \qquad A = -\frac{\beta}{2}\frac{R^4}{r^2}\,\sigma_3\,. \tag{2.19}$$

This matching also requires a simple gauge transformation on the potentials.

Near the domain wall, it is sensible to express the local charges of the BPS algebra in terms of the radius R. For example, the central charges corresponding to D0-branes and the F-strings are $\gamma_0 R^2$ and $\gamma_s R^2$ respectively, which can be clearly seen by considering the integer charges as functions of moduli times the central charges:

$$\begin{aligned}
n_0 &= Q_0 \cdot \Big(\tilde{g}_s \frac{\pi^2}{2}\,(\alpha')^{1/2} \tilde{R}_9 \tilde{G}_6^{-1}\Big) \;=\; \gamma_0 R^2 \cdot \Big(g_s \frac{\pi^2}{2}\,(\alpha')^{1/2} R_9 G_6^{-1}\Big) \\
n_s &= Q_s\,\frac{\pi^2\alpha'}{2\tilde{G}_6} \;=\; \gamma_s R^2 \cdot \Big(\frac{\pi^2\alpha'}{2G_6}\Big).
\end{aligned} \tag{2.20}$$

This redshift of the central charges plays an important role in the attractor phenomena of BPS black holes [20], but since we always cut off our solution at non-zero R the charges never reach any fixed points.

2.3 The Outside Part II: The Three-Charge BMPV Rotating Extremal Black Hole

The two-charge solution represents the minimal asymptotically flat solution that can serve as a causal outside extension of the Gödel universe, with – as we will see in the next subsection – a simple smeared supertube playing the role of the domain wall between the two geometries. Things become even more interesting when we consider outside solutions with more charges, such as the three-charge BMPV black hole. The matching will still be possible, and the domain wall required in the process will exhibit some interesting novel properties.

In this subsection, we start with the following, slightly more generic candidate for the outside metric,

$$\begin{aligned}
ds^2 &= -\tilde{U}^{-1}\tilde{V}^{-1/2}\tilde{W}^{-1/2}(d\tilde{t} + \frac{J}{4\tilde{r}^2}\,\sigma_3)^2 + \tilde{U}^{-1}\tilde{V}^{1/2}\tilde{W}^{1/2} d\tilde{y}^2 \\
&\qquad\qquad + \tilde{V}^{1/2}\tilde{W}^{1/2}\sum_{i=1}^{4}(d\tilde{x}^i)^2 + \tilde{V}^{1/2}\tilde{W}^{-1/2}\sum_{i=5}^{8}(d\tilde{x}^i)^2, \\
B_2 &= -\tilde{U}^{-1}(d\tilde{t} + \frac{J}{4\tilde{r}^2}\,\sigma_3)\wedge d\tilde{y} + d\tilde{t}\wedge d\tilde{y}, \qquad e^{\Phi} = \tilde{g}_s\,\tilde{U}^{-1/2}\tilde{V}^{3/4}\tilde{W}^{-1/4}, \\
\tilde{g}_s C_1 &= -\tilde{V}^{-1}(d\tilde{t} + \frac{J}{4\tilde{r}^2}\,\sigma_3) + d\tilde{t}, \\
\tilde{g}_s C_3 &= \frac{1}{4}\,Q_4 \cos\theta\, d\phi\wedge d\psi\wedge d\tilde{y} + \tilde{U}^{-1} d\tilde{t}\wedge d\tilde{y}\wedge \frac{J}{4\tilde{r}^2}\,\sigma_3
\end{aligned} \tag{2.21}$$

with

$$\tilde{U}(\tilde{r}) = 1 + \frac{Q_s}{\tilde{r}^2}, \qquad \tilde{V}(\tilde{r}) = 1 + \frac{Q_0}{\tilde{r}^2}, \qquad \tilde{W}(\tilde{r}) = 1 + \frac{Q_4}{\tilde{r}^2}. \tag{2.22}$$

As mentioned before, this three-charge BMPV solution is a special case of the solutions [17] of Gauntlett, Myers and Townsend with four supersymmetries; we have now set only one of their M5-brane charges to zero. The sign of the additional D4-branes (descendants of the added M5-branes) determines which of the $\tilde{r}$-dependent Killing spinors remain, those proportional to ε^{+-+} or those proportional to ε^{+++}, thus reducing the number of supersymmetries.

The addition of the third charge to the metric has an important effect on the physics of the solution. The critical radius for CTC's, with a zero-area three-sphere, is now located at

$$4\,(\tilde{r}_c^2 + Q_4)\,(\tilde{r}_c^2 + Q_0)\,(\tilde{r}_c^2 + Q_s) = J^2, \tag{2.23}$$

with $\tilde{r}_c^2$ positive if $J^2 > 4Q_4Q_0Q_s$, and negative otherwise. The case of $J^2 > 4Q_4Q_0Q_s$ describes an "over-rotating black hole" [12, 13, 21], which in fact is not a black hole at all; instead, $\tilde{r} = 0$ is a naked singularity, surrounded by a naked region of CTCs. This case is very similar to the spinning two-charge solution that we discussed in the previous subsection. If $J^2 \leq 4Q_4Q_0Q_s$, the solution describes an honest black hole whose horizon is located at $\tilde{r} = 0$. The region with CTCs still exists, but is now cloaked by the horizon, which is a squashed three-sphere with a finite sized horizon area

$$\mathcal{A}_H = \pi^2 (Q_0Q_4)^{1/4} Q_s^{-1/2} \sqrt{4Q_4Q_0Q_s - J^2}. \tag{2.24}$$

Once again we will need to perform a coordinate transformation to near-tube coordinates which match with the inside solution. Consider placing the domain wall at $\tilde{r} = \tilde{R}$, (i.e., outside the horizon if the solution is under-rotating). To match the coordinate systems, we define parameters χ_i, much like in the two-charge case:

$$\chi_s \equiv \tilde{U}(\tilde{R}) = 1 + \frac{Q_s}{\tilde{R}^2}, \qquad \chi_0 \equiv \tilde{V}(\tilde{R}) = 1 + \frac{Q_0}{\tilde{R}^2}, \qquad \chi_4 \equiv \tilde{W}(\tilde{R}) = 1 + \frac{Q_4}{\tilde{R}^2}. \tag{2.25}$$

Also, just as in the two-charge case, we define new parameters γ_i

$$\gamma_s = 1 - \chi_s^{-1}, \qquad \gamma_0 = 1 - \chi_0^{-1} \qquad \gamma_4 = 1 - \chi_4^{-1}. \tag{2.26}$$

All γ_i belong to $[0, 1]$ in order to ensure that the domain wall is placed outside or at the horizon.

Unlike in the two-charge case, however, we have to take into account the possibility of putting our domain wall behind the horizon. Even though $\tilde{r}$ itself is no longer a good coordinate behind the horizon, $\rho \equiv \tilde{r}^2$ is a good function there. In fact, ρ itself can serve as a coordinate that replaces $\tilde{r}$ behind the horizon; another particularly nice coordinate

that extends through the horizon is the radius r' of the three-sphere in the string frame, $r' \equiv \tilde{r}\tilde{V}^{1/2}\tilde{W}^{1/2}$. In the region behind the horizon, our function $\tilde{r}^2$ (as a function of either ρ or r') takes negative values bound from below by the value of the smallest of the three charges (which we denote by $\widehat{Q}$),

$$\tilde{r}^2 \geq -\widehat{Q} \equiv -\min(Q_0, Q_4, Q_s), \tag{2.27}$$

with $\tilde{r}^2 = -\widehat{Q}$ describing the singularity.

The possibility of placing the domain wall behind the horizon (but above the singularity) now extends the range of our original γ parameters given in (2.26) to satisfy

$$\gamma_i \leq \frac{Q_i}{Q_i - \widehat{Q}.} \tag{2.28}$$

If the domain wall is placed behind the horizon, the matching of the induced metric against the Gödel interior requires the opposite sign of the Gödel vorticity parameter β. This is in accord with the intuition that the region behind the horizon of the BMPV black hole contributes a negative amount to the total (positive) value of the angular momentum of the black hole. Since one of the main points of this paper is the possibility of resolving *naked* closed timelike curves, we will restrict our attention for the rest of the paper to the domain walls placed above the horizon, i.e., we restrict $\gamma_i \in [0,1]$, and $\tilde{R}^2 \geq 0$.

Similarly to the two-charge case, we define a smooth coordinate system across the domain wall using simple rescalings,

$$\begin{aligned} \tilde{t} &= \chi_s^{1/2}\chi_0^{1/4}\chi_4^{1/4}\, t, & \tilde{y} &= \chi_s^{1/2}\chi_0^{-1/4}\chi_4^{-1/4}\, y, \\ \tilde{x}^i|_{i=1..4} &= \chi_0^{-1/4}\chi_4^{-1/4}\, x^i, & \tilde{x}^i|_{i=5..8} &= \chi_0^{-1/4}\chi_4^{+1/4}\, x^i. \end{aligned} \tag{2.29}$$

Matching the volumes of T^4, S^1 and S^3, as well as the squashing of the S^3, gives

$$\tilde{V}_4 = \chi_0^{-1}\chi_4 V_4, \quad \tilde{R}_9 = \chi_s^{1/2}\chi_0^{-1/4}\chi_0^{-1/4} R_9, \quad \tilde{R} = \chi_0^{-1/4}\chi_4^{-1/4} R, \quad J = \chi_s^{1/2}\chi_0^{-1/4}\chi_4^{-1/4}\, 2\beta R^4. \tag{2.30}$$

The asymptotic string coupling and six-dimensional Newton's constant in terms of their values at the domain wall are

$$\tilde{g}_s = g_s\chi_s^{1/2}\chi_0^{-3/4}\chi_4^{1/4}, \qquad \tilde{G}_6 = G_6\chi_s\chi_0^{-1/2}\chi_4^{-1/2}. \tag{2.31}$$

In the near-tube coordinates, the metric now looks like eq. (2.21) but with new harmonic functions in the metric:

$$U = 1 + \gamma_s(\frac{R^2}{r^2} - 1), \quad V = 1 + \gamma_0(\frac{R^2}{r^2} - 1), \quad W = 1 + \gamma_4(\frac{R^2}{r^2} - 1), \quad A = -\frac{\beta}{2}\frac{R^4}{r^2}\,\sigma_3\,. \tag{2.32}$$

We use a series of simple gauge transformations to match the gauge potentials at $r = R$ with those of the Gödel solution.

The charges of the BPS algebra at the domain wall are now $\gamma_0 R^2, \gamma_s R^2$ and $\gamma_4 R^2$ as can be seen, for example, by comparing expressions for the number of D4-branes at infinity and at the tube

$$n_4 = Q_4 \cdot \tilde{g}_s^{-1} (\alpha')^{-3/2} \tilde{R}_9 \; = \; \gamma_4 R^2 \cdot g_s^{-1} (\alpha')^{-3/2} R_9. \tag{2.33}$$

If we want to place our domain wall at the horizon, we must take $\tilde{R} \to 0$ and thus all the $\gamma_i \to 1$. As expected, all the central charges converge to same fixed value [20]. Note that in this case, the transformation between our two coordinate systems – as given in eq. (2.29) – becomes singular, reflecting the infinite redshift from the domain wall to asymptotic infinity. We can still define the near-tube coordinate system, though, by extracting the value for R^2 from the attractor for the central charges of the BPS algebra. We then get β from

$$\beta = \frac{1}{2} J R^{-4}, \tag{2.34}$$

and write the metric for a domain wall at the horizon using the harmonic forms in eq. (2.32). In Section 5, we will see how this limit on the parameters γ_i is the U-dual of the Maldacena limit [22] of the D1/D5/KK system.

3 Bridging the Divide: the Domain Wall

Now that we have two outside solutions in hand which we would like to match to the five-dimensional Gödel universe, we need to see what kind of domain wall stress-energy the Israel matching conditions will impose on us. We will then demonstrate for each case how a particular smeared supertube, along with possible D4-branes, satisfies this conditions, and match all the relevant parameters.

3.1 Matching the Solutions

Given the inside and outside solutions in the previous section, let us compute the Israel matching conditions through a domain wall at $r = R$ using the perpendicular unit norm co-vector:

$$\hat{n}_\nu dx^\nu = \sqrt{g_{rr}} dr. \tag{3.1}$$

The extrinsic curvature is easy to compute since there are no $g_{r\mu}$ cross-terms in any of the metrics:

$$K_{\mu\nu} = \nabla_\mu \hat{n}_\nu = \frac{1}{2} (g_{rr})^{-\frac{1}{2}} \partial_r g_{\mu\nu}. \tag{3.2}$$

For the metric (2.4) this gives:

$$K^-_{\mu\nu}dx^\mu dx^\nu|_{r=R} = -(\beta R\,\sigma_3)(dt+\beta\,\frac{R^2}{2}\,\sigma_3)+\frac{R}{4}(\sigma_1^2+\sigma_2^2+\sigma_3^2) \tag{3.3}$$

The trace of this extrinsic curvature is:

$$K^-|_{r=R} = h^{\mu\nu}\,K^-_{\mu\nu}|_{r=R} = \frac{3}{R} \tag{3.4}$$

For the outside solutions we need to be careful, as the metrics so far have been string metrics. The Israel matching condition involves the Einstein metric which for the first outside solution (with two charges) is:

$$ds_E^2 = -U^{-3/4}V^{-7/8}(dt-A)^2+U^{-3/4}V^{1/8}dy^2+U^{1/4}V^{1/8}\sum_{i=1}^{8}(dx^i)^2. \tag{3.5}$$

This gives an extrinsic curvature at $r=R$ of the form (keeping in mind $U(R)=V(R)=1$):

$$\begin{aligned} K^+_{\mu\nu}\,dx^\mu dx^\nu|_{r=R} &= (3/8\,U'+7/16\,V')(dt-A)^2+\beta R\,\sigma_3(dt-A) \\ &+(-3/8\,U'+1/16\,V')\,dy^2+\ (1/8\,U'+1/16\,V')\sum_{i=1}^{8}(dx^i)^2 \\ &+\frac{R}{4}(\sigma_1^2+\sigma_2^2+\sigma_3^2). \end{aligned} \tag{3.6}$$

which has trace ($\sigma_3(dt-A)$ is traceless)

$$\begin{aligned} K^+|_{r=R} = h^{\mu\nu}\,K^+_{\mu\nu} &= (-3/8\,U'-7/16\,V')+(-3/8\,U'+1/16\,V') \\ &+\ 7(1/8\,U'+1/16\,V')+\frac{3}{R} \\ &= (1/8\,U'+1/16\,V')+\frac{3}{R}. \end{aligned} \tag{3.7}$$

The Israel matching condition now tells us that the domain wall stress tensor has to be:

$$\begin{aligned} (8\pi G_{10})\,S^I_{\mu\nu} &= \gamma_{\mu\nu}dx^\mu dx^\nu \\ &= \Big(K^+_{\mu\nu}-K^-_{\mu\nu}-h_{\mu\nu}(K^+-K^-)\Big)\,dx^\mu dx^\nu \\ &= \frac{1}{2}(U'+V')(dt-A)^2-\frac{1}{2}U'\,dy^2+2(\beta R\,\sigma_3)(dt+\beta\,\frac{R^2}{2}\,\sigma_3) \\ &= -\frac{\gamma_s+\gamma_0}{R}\left(dt+\frac{\beta R^2}{2}\sigma_3\right)^2+2\beta R\left(dt+\frac{\beta R^2}{2}\sigma_3\right)\sigma_3+\frac{\gamma_s}{R}\,dy^2. \end{aligned} \tag{3.8}$$

A similar set of computations ensures that the jumps in the field strengths associated with the B-field and the C-fields (let's call them δH_3, δG_2 and δG_4) is consistent with the charges of the domain wall. Here, we will illustrate just one of them, the other matchings being a

straightforward extension of the same exercise. Thus, the correct definition for the four-form field strength is:

$$G_4 = dC_3 - dB_2 \wedge C_1 = -\,U^{-1}V^{-1}\,dy \wedge dA \wedge (dt - A). \tag{3.9}$$

The equation of motion for G_4 is:

$$d * G_4 + H \wedge G_4 = 2\kappa^2 J_7 \tag{3.10}$$

When integrated across the domain wall, the $H \wedge G_4$ term has insufficient derivatives (i.e., only step-function distributions, no delta functions) to contribute and we are left with:

$$\delta(*G_4) = 16\pi\, G_{10}\, \frac{\delta}{\delta C_3}\, S_{DW}. \tag{3.11}$$

An explicit evaluation of the jump $\delta(*G_4)$ in this case reveals that our domain wall gives rise to a D2-brane dipole moment, induced from an effective local D2-brane charge of the wall whose value turns out to be

$$N_{2,\mathrm{eff}} = \beta\,(4\pi\, G_{10})^{-1}\left(\tau_2^{-1}\,\pi R^2\, V\right). \tag{3.12}$$

We will see in the next subsection how this prediction for the value of the local D2-brane charge carried by the domain wall matches exactly with the microscopic properties of the constituents that sustain the domain wall.

For the second outside solution, the three-charge rotating extremal black hole, an equivalent calculation of the matching condition also yields a very simple form of the effective stress-energy tensor of the domain wall,

$$(8\pi G_{10})\, S^{II}_{\mu\nu} = -\frac{\gamma_s + \gamma_0 + \gamma_4}{R}\left(dt + \frac{\beta R^2}{2}\sigma_3\right)^2 + 2\beta R\left(dt + \frac{\beta R^2}{2}\sigma_3\right)\sigma_3 + \frac{\gamma_s}{R}\,dy^2 + \frac{\gamma_4}{R}\,ds^2_{T^4}. \tag{3.13}$$

We see that the total stress-energy tensor of the wall splits into a sum of individual contributions due to the three individual charges and to the angular momentum. This fact becomes even more transparent in the contravariant form of the stress-energy tensor, with both indices raised.

3.2 The Stress-Energy of Supertubes

Here we will compute the stress-energy tensor for a supertube extended along the directions t, y, and wrapped on the Hopf fiber ϕ (with $\sigma_3 = d\phi + \cos\theta d\psi$). The supertube wrapping the Hopf fiber at some fixed location on the base S^2 would violate some of our mandatory

$U(1)_L \times SU(2)_R$ symmetry. The key to restoring this symmetry is that our supertube will be *smeared* along the compact internal dimensions *and* along the $\mathbf{S}^2$ base of the Hopf-fibered S^3. Before smearing, this supertube is located at a point in the base S^2 and in T^4, and carries an induced metric and gauge fields of the form:[3]

$$\begin{aligned} ds^2_{ind} &= \mathcal{G}_{ab}\, d\xi^a\, d\xi^b = -(dt + \beta \frac{R^2}{2} d\phi)^2 + \frac{R^2}{4}\, d\phi^2 + dy^2 \\ \mathcal{B}_2 &= -\beta \frac{R^2}{2} d\phi \wedge dy,\ \mathcal{C}_1 = -\beta \frac{R^2}{2} d\phi,\ \mathcal{C}_3 = dt \wedge dy \wedge \beta \frac{R^2}{2} d\phi \end{aligned} \tag{3.14}$$

The gauge invariant modified field strength on the brane is:

$$\mathcal{F} = F - \mathcal{B}_2 = E dt \wedge dy + \mathcal{B}\, dy \wedge d\phi, \tag{3.15}$$

where $\mathcal{B} = (\frac{B}{2} - \beta \frac{R^2}{2})$. With this normalization, it is easy to check that the action for an individual supertube before smearing takes its canonical form familiar from flat space, including the proper overall normalization of the action. If we define Π as the canonical conjugate of E, this observation in particular implies that our quantities Π, E, B and R satisfy relations familiar from flat space, such as $\Pi\, B = R^2$ (assuming that the equations of motion are satisfied).

The action for N_2 supertubes smeared over the base S^2 and T^4 is:

$$S = -\mathcal{N} \int e^{-\Phi} \sqrt{-\det(\mathcal{G} + \mathcal{F})} - \mathcal{N} \int (\mathcal{C}_3 + \mathcal{C}_1 \wedge \mathcal{F}). \tag{3.16}$$

where we have a included density factor which takes into account the smearing of our solution over the volume of $S^2 \times T^4$:

$$\mathcal{N} = N_2 \tau_2 \cdot \left(\pi R^2\, V\right)^{-1}. \tag{3.17}$$

The second term is topological and will not contribute to the stress-energy tensor, while the first term can be re-written as

$$S' = -\mathcal{N} \int \sqrt{-\mathcal{G}} \sqrt{(1 + \frac{1}{2}\, \mathcal{F}^2)} \tag{3.18}$$

which upon variation gives

$$S^{\mu\nu} = \left(\frac{\partial x^\mu}{\partial \xi^a}\right)\left(\frac{\partial x^\nu}{\partial \xi^b}\right) \cdot \left(\frac{2}{\sqrt{-\mathcal{G}}} \frac{\delta S'}{\delta \mathcal{G}_{ab}} = \mathcal{N}\left(\sqrt{(1 + \mathcal{F}^2/2)}\, \mathcal{G}^{ab} + (1 + \mathcal{F}^2/2)^{-\frac{1}{2}} F^{ac} F_c{}^b\right)\right). \tag{3.19}$$

To compare with eq. (3.8) we need the domain wall stress-tensor in its covariant form, so we use the full metric $g_{\mu\nu}$ to lower its two indices. After solving the equations of motion as in

[3]On the worldvolume of the D2-brane, we use the obvious choice of induced coordinates $(\xi_0, \xi_1, \xi_2) = (t, \phi, y)$. Note, however, that the range of the periodic coordinate ϕ is $[0, 4\pi)$, which leads various factors of two compared to the conventional supertube worldvolume coordinates, such as those used in [5].

the probe calculation of [6], we find that – just as in flat space – $E = 1$, and $\sqrt{(1 + \mathcal{F}^2/2)} = B/R$. These facts allow us write the stress-energy in the following simple form,

$$-\frac{\mathcal{N}}{R}(B + \Pi)\left(dt + \frac{\beta R^2}{2}\sigma_3\right)^2 + \mathcal{N}R\left(dt + \frac{\beta R^2}{2}\sigma_3\right)\sigma_3 + \frac{\mathcal{N}\Pi}{R}dy^2. \tag{3.20}$$

If we compare with the stress tensor (3.8) from the Israel matching conditions,

$$-\frac{1}{R}(\gamma_s + \gamma_0)\left(dt + \frac{\beta R^2}{2}\sigma_3\right)^2 + 2\beta R\left(dt + \frac{\beta R^2}{2}\sigma_3\right)\sigma_3 + \frac{\gamma_s}{R}dy^2, \tag{3.21}$$

we get that

$$(8\pi G_{10})\,\mathcal{N} = 2\beta, \qquad \gamma_0 = 2\beta\, B, \qquad \gamma_s = 2\beta\,\Pi. \tag{3.22}$$

Plugging this expression for β back into the formula for the expected local D2-brane charge of the domain wall given in Eq. (3.12), we get as a nice consistency check that

$$N_{2,\text{eff}} = N_2. \tag{3.23}$$

3.3 Adding D4-branes

So far we have succeeded in matching the first outside solution, the two-charge reduction of the BMPV black hole, with the Gödel solution using the supertube described above. The situation with a three-charge black hole outside is more complicated. Once we introduce D4-brane charge, the outside solution with just a D2-brane dipole now develops NS5 and D6 dipoles. There is now a wide variety of D0/D4/F1 bound states which explicitly exhibit these dipoles using local D6/D2/NS5 charges, much as the supertube carries the local D2-brane charge [23]. We start with the simplest generalization of the supertube: a dust of T^4-wrapped D4-branes smeared along and co-moving with the supertube.

In order to proceed, we will use the fact that the addition of the D4-branes only breaks half of the supertube supersymmetry. This means that the total stress energy of the domain wall is controlled by the BPS algebra: it is just the sum of (3.20) with that of plain vanilla D4-branes smeared along the y-direction and the three-sphere of radius R. The stress energy for the D4-branes is

$$n_4\,(\tau_4^{-1}\cdot 2\pi^2 R^3 \cdot 2\pi R_9)^{-1}\left\{-\left(dt + \frac{\beta R^2}{2}\sigma_3\right)^2 + \sum_{i=5}^{8}(dx^i)^2\right\} \tag{3.24}$$

The total stress-energy will match if in addition to eq. (3.22), we have

$$n_4 \;=\; \gamma_4\, g_s\,\frac{8\pi^6(\alpha')^{5/2}R^2 R_9}{G_{10}} = \gamma_4 R^2 \frac{R^9}{g_s l_s^3}, \tag{3.25}$$

which is exactly what we have in eq. (2.33), as expected from the BPS condition that mass equals charge. Thus, the microscopic details of the bound state are largely irrelevant for the macroscopic behavior of the domain wall. For any general domain wall with the same amount of supersymmetry, made up of any collection of D0/F1/D4 bound states, the stress-energy always splits into the form,

$$S^{\mu\nu} = S_0^{\mu\nu} + S_s^{\mu\nu} + S_j^{\mu\nu} + S_4^{\mu\nu} \tag{3.26}$$

manifested already in Eq. (3.13). The particular microscopic characteristics of the bound will only determine a macroscopic relation between the angular momentum and other macroscopic parameters of the configuration. Only this macroscopic condition is needed to ensure the validity of the Israel matching conditions.

4 Microstates and the Asymptotic Bulk Fields

We now want to understand our domain wall matching in the context of supertube microstates. More generally, we aim at a deeper understanding of the relationship between various detailed microstates which come from bound states of F1,D0 and D4-branes, and the asymptotic bulk fields which they source. The first step is to count configurations with large $U(1)_L$ angular momentum j_L. We can facilitate this process by counting U-dual states with the same angular momentum.

We first examine the case when all the angular momentum of a putative three-charge black hole ground state is carried by just one type of charged component. For example, suppose we U-dualize such that all the angular momentum is carried by wrapped F1-strings. In order to transform under the same super-conformal algebra as the black hole, these must be in the RR ground state. These states transform in the $(\mathbf{1},\mathbf{1})$, $(\mathbf{3},\mathbf{1})$, $(\mathbf{1},\mathbf{3})$ and $(\mathbf{2},\mathbf{2})$ of $SU(2)_L \times SU(2)_R$. This implies that the maximum $U(1)_L$ charge carried by n_i microstate components alone is

$$j_L = 2n_i, \text{ with } \quad j_R = 0. \tag{4.1}$$

Next we consider cases where angular momentum is carried by bound states of two charges (n_i, n_j). We take as our canonical example the D1/KK(momentum) system ($n_i = n_1, n_j = n_{KK}$). One important characteristic of this system comes from the zero modes of the D1-D1 open strings. To get the maximal angular momentum, we wind a single D1 string n_1 times, and then use the fact that the mimimum KK-momentum for the left-moving strings is now $1/n_1 R$ to carry this KK-momentum with $n_1 n_{KK}$ open strings. Each of these has angular momentum in the $(\mathbf{1},\mathbf{1})$ or $(\mathbf{2},\mathbf{2})$ if it is bosonic or in the $(\mathbf{1},\mathbf{2})$ or $(\mathbf{2},\mathbf{1})$ if it is fermionic.

This means that the maximum value we can achieve for j_L is now

$$j_L = n_1 n_{KK} = n_i n_j. \tag{4.2}$$

For such a state, bosonic and fermionic statistics imply that j_R must take the value $n_i n_j$, $n_i n_j - 1$, or $n_i n_j - 2$. With two fermionic bound states, we can make j_R very close to zero. In the maximal angular momentum case above, we should point out that the U-dual D1/D5 system maximally fragments into $n_i n_j$ string bits, each carrying the maximum amount of angular momentum (for more details see [24]).

Finally, we consider the angular momentum carried by three bound charges with four supersymmetries. Our canonical choice is now the D1/D5/KK system excited so as to carry as much angular momentum as possible. In this case, one long string in the RR sector[25], with length $2\pi R_9 n_1 n_5 n_k$, carries all the angular momentum through left-handed fermionic strings with one unit of the $U(1)_L$ charge. The right handed sector has $U(1)_R$ charge ± 1. In the limit where one of the charges, such as n_k is much larger than the product of the other two, the maximum angular momentum is [12]

$$j_l = 2\sqrt{n_i n_j n_k}, \tag{4.3}$$

with $j_R = \pm 1$.

We finish this summary by remarking on the effectiveness of various bound states at carrying angular momentum. If we start by carrying all the angular momentum with two charges, we can compare eqs. (4.1) and (4.2):

$$n_i n_j > 2n_i + 2n_j \quad \text{for} \quad n_i, n_j > 3. \tag{4.4}$$

This inequality means that different sets of multiple charges can almost always carry more angular momentum by binding together than they could alone. When we move to three charges, this binding advantage no longer seems to apply in all cases. For example, if we take one charge much larger than the others

$$n_3 \gg n_1 n_2 \tag{4.5}$$

then we can increase the overall j_L by placing all of the angular momentum in the third charge since

$$2n_3 = 2\left(\sqrt{n_3}\right)^2 \gg 2\sqrt{n_1 n_2 n_3}. \tag{4.6}$$

4.1 Interpretation of Matching Part I: The Two-Charge Domain Wall

With the context of various black hole microstates now in mind, we can now interpret the results of our matching for the first outside solution, the two-charge reduction of the rotating

black hole. Apart from the proper matching of the D0-brane and F1-string charges we get two important results which relate the microscopics of the domain wall with the parameters of the bulk solution.

The first result comes from the internal dynamics of the supertube, $R^2 = \Pi B$, plus the matching of D0 and F1 charges. It yields

$$\gamma_s\,\gamma_0 = 4\beta^2 R^2 \tag{4.7}$$

We note immediately that by definition we have $\gamma_0, \gamma_s < 1$. This inequality combined with the equation above directly implies that $R < 1/2\beta$. Hence, we never have closed timelike curves inside the domain wall (and thus no CTCs outside either)! The redshifting of central charges, $\gamma_i = Q_i/(Q_i+\tilde{R}^2)$, allows us to extract one more piece of information out of eq. (4.7): the relationship between the principal macroscopic dimensionful quantities,

$$Q_s\,Q_0\,\tilde{R}^2 = J^2. \tag{4.8}$$

Comparing with eq. (2.13), we can check that $\tilde{R}$ is clearly larger than the critical radius r_c. Here, the squared radius of the domain wall (related to the D2 dipole, as we will see) plays much the same role in the macroscopic charge relations as the D4-brane charge does for the three-charge BMPV black hole. In fact, in the next subsection when we add in spectator D4-brane charge, the domain wall radius squared and the D4-brane charge will appear additively.

The second result from our matching data, eq. (3.22), is that:

$$\beta R^2 = \,4N_2 G_6 \tau_2, \tag{4.9}$$

If we combine this relation with eq. (4.7) we get two separate equations for the integral value of the angular momentum. The first one,

$$j = \frac{\pi J}{4\tilde{G}_5} = \frac{\pi}{2\tilde{G}_5}\beta\tilde{R}^4\chi_s^{1/2}\chi_0^{3/4} = n_0\,n_s/N_2 = N_2\left(\frac{n_0}{N_2}\right)\left(\frac{n_s}{N_2}\right)\,, \tag{4.10}$$

is exactly like the expression for a supertube with rank-two angular momentum in flat space. According to eq. (4.2), this is also the maximum angular momentum allowed for any microstate which comes from N_2 bound-states of n_0/N_2 D0-branes and n_s/N_2 fundamental strings. The second equation is

$$j = \frac{\pi J}{4\tilde{G}_5} = \frac{\pi^2 J\tilde{R}_9}{2\tilde{G}_6} = N_2\,\tau_2\,4\pi^2 R_9\,R^2, \tag{4.11}$$

which shows that j is the sum of angular momenta from the N_2 $D2$-branes with identical dipole moments.

To recap, the classical BMPV black hole solution with only two charges Q_0, Q_s and nonzero angular momentum J has a naked singularity at $r = 0$ and exhibits closed time-like curves in the vicinity of the singularity (and consequently throughout the entire spacetime). For a sufficiently small, yet macroscopic angular momentum we find that the classical solution can be naturally truncated at a fixed radius by a smeared supertube, removing all closed time-like curves. An interesting thing to note here is that the microstate condition $jN \leq n_0 n_s$, gives a bound on the asymptotic charges J, Q_0, Q_s which depends on the moduli $\tilde{g}_s$ and $\tilde{V}$:

$$J \leq \frac{Q_0 Q_s}{8\tilde{\tau}_2 \tilde{G}_6}. \tag{4.12}$$

Of course, here we have only described the resolution of the BMPV solution for the case $Nj_L = n_0 n_s$. For smaller amounts of angular momentum, a circular supertube carrying all the charge will not work. To remedy this, we can either increase the amount of charge free (unbound) from the supertube, or change the shape of the supertube (which reduces its angular momentum [26, 27, 28]). D0-branes and F1-strings are BPS states with compatible supersymmetries, so they can be placed anywhere and they will feel no force. For sufficiently large numbers of these free charges, this means we could form supersymmetric two-charge Gödel black holes inside our domain wall. On the other hand, we can also make ripples on the D2-brane supertube which will lower its angular momentum but asymptotically only contribute to higher D2-brane moments. Both configurations will quickly start to look like the appropriate BMPV black hole asymptotically. In order to preserve an exact $SU(2)_R$, all these configurations still need to be smeared. Only the un-smeared rippled supertubes are proper boundstate microstates with no loose charges; they are U-dual to the black hole hair of Mathur and Lunin [29, 30, 31, 32, 33, 34].

We end our discussion of the two-charge case by pointing out that in addition to our smearing, the asymptotic solution is further coarse-grained by the fact that it only relays classical information about angular momentum. An observer far enough away will not be able to distinguish between our domain wall and one with $m_R \ll n_0 n_s$. It is possible that the 3-sphere area of our smeared supertube is related to the number of D0/F1 bound states which are identical under this further coarse graining.

4.2 Interpretation of Matching Part II: The Three Charge Domain Wall

We now proceed to consider the domain wall match between the three-charge BMPV black hole and our five-dimensional Gödel universe. Contrasted with the the two-charge case, there is now a greater variety of D0/F1/D4 bound state micro-states which we can smear to preserve an exact $U(1)_L \times SU(2)_R$ R-symmetry. This implies a whole zoo of possible objects

uniformly distributed on a squashed three-sphere which we could use as a domain wall. We have given the exact details for only one case, a D0-F1 supertube with an unbound dust of T^4-wrapped D4-branes. In this subsection, we will show how the details of this specific choice of microstate feed backs into the parameters of the bulk solution. We will explore what a more general domain wall implies for these parameters in the next subsection.

We have chosen to put our domain wall outside or on the horizon, so we again have $\gamma_0, \gamma_s, \gamma_4 \leq 1$. Since we have not modified the supertube, and are dealing with BPS states, the matching conditions yield exactly the same form as before for the parameter β, and we still have an "equation of state" for the supertube of the form $4\beta^2 R^2 = \gamma_0 \gamma_s$. However, the redshift factors are now different: this changes the equation relating the macroscopic dimensionful quantities in the asymptotic coordinate system. Indeed, now

$$J^2 = Q_s Q_0 \tilde{R}^2 \chi_4^{-1} = \gamma_4^{-1} \cdot Q_s Q_0 Q_4 = Q_s Q_0 \,(Q_4 + \tilde{R}^2). \tag{4.13}$$

Once again, the domain wall radius is such that no closed time-like curves appear. For a more general three charge supertube, this equation of state will have to be modified to reflect the details of the bound state which carries the angular momentum.

The second item we get from matching at the domain wall is the value of βR^2 in terms of the moduli at the domain wall. This just comes from matching the stress-energy tensor: for our choice of the domain wall (smearing N_2 supertubes plus $D4$-dust) we have

$$\beta R^2 = \, 4 N_2 \, G_6 \, \tau_2. \tag{4.14}$$

This give the following two equations for j

$$n_0 n_s / N_2 = j = N_2 \, 4\pi^2 \tilde{R}_9 \, \tilde{\tau}_2 \, \tilde{R}^2 = N_2 \, \tau_2 \, 4\pi^2 R_9 \, R^2, \tag{4.15}$$

reflecting that fact that all the angular momentum is being carried by the D0/F1 bound-state D2-dipoles. The D4-branes carry no extra angular momentum.

Going back to eq. (4.13), we see that for a dipole of sufficiently large radius, the three-charge BMPV black hole will be over-rotating. Once again, we find that our domain wall shell acts as a guardian of chronology, cutting off the chronology-violating region of the classical solution, and replacing it with a causal core.

It may also be instructive to consider under which conditions the black hole will be under-rotating. (For simplicity's sake, we will now assume $N_2 = 1$.) In order that the total angular momentum is small enough to guarantee the existence of a horizon, eq. (4.13) shows that we must have:

$$\gamma_4 > 1/4. \tag{4.16}$$

In terms of integral charges, this implies

$$n_4 > n_0 n_s / 4\,. \tag{4.17}$$

For such large n_4 the bound on the angular momentum for a bound-state of n_0 D0-branes and n_s F1-strings with n_4 wrapped D4-branes is

$$j \leq 2\sqrt{n_0 n_s n_4} \tag{4.18}$$

so it is possible for this bound-state to carry at least as much momentum as a supertube of bound D0's and F1's alone.

These conditions on over-rotation vs. under-rotation are consistent with our microscopic understanding of black holes for the following reason. If we were to perturb our shell with a little bit of extra energy, two things could happen. If n_4 is large enough, the D4-branes can bind with the branes in the supertube and the whole object becomes a microstate of an under-rotating black hole since these are D4/D0/F1 bound-states. On the other hand, if n_4 is too small, any black hole we try to form will leave a shell of rotating matter behind. The asymptotic fields will be that of an over-rotating black hole, but chronology will still be saved by a domain wall. These will be solutions corresponding to Gödel black holes [7, 8, 9, 10, 11] separated from an over-rotating BMPV black hole by a domain wall.

4.3 Other Domain Walls

In [35], another possible mechanism was offered for preventing the creation of over-rotating black hole. When the author of [35] tried to bring in extra angular and KK-momentum on adiabatically collapsing shell, she discovered an enhançon-like effect which encouraged the shell to remain at a radius[4] which, after a T-duality transformation mapping D1/D5/KK to D0/D4/F1, solves the equation

$$J^2 = 4Q_s(Q_0 + \tilde{R}_D^2)(Q_4 + \tilde{R}_D^2) \tag{4.19}$$

In this instance, the bound on the angular momentum is once again circumvented by sequestering proposed extra mass and angular momentum in a shell outside the horizon. Even if we place all the charge on this shell, the solution inside is no longer a Gödel solution, it is a homogenous fluxbrane solution (a different T-duality takes this to flat space).

In the context of our work, we see that this time the angular momentum is carried by a single charge n_s. The maximum groundstate $U(1)_L$ spin carried by these charges happens

[4] We propose that the resulting sphere should be called a Dyson sphere.

when $j_L = 2n_s, j_R = 0$. A little hard work gives us equations very similar to those from a supertube domain wall:

$$\beta^2 R^2 = \gamma_s, \qquad 2n_s = j = \frac{\pi R_9^2}{\alpha' G_6} R^4. \tag{4.20}$$

Note that now j grows with the fourth power of R. A U-duality transformation gives us similar monopole shell resolution for D0's and D4's. In those cases, the angular momentum comes from the ground-state zero modes

As opposed to the supertube dipoles, the enhançons or Dyson spheres only appear with positive $\tilde{R}^2$ for over-rotating three-charge BMPV black holes. Depending on the size of the single charge in question, though, they will truncate the solution either inside or outside the radius at which our supertubes truncate it. It would be interesting to understand the physical meaning of the corresponding cross-over point.

Apart from these individually spinning-charge domain walls, there are of course other dipole domain walls which come from U-duality. For example, a simple T-duality transformation takes the D2-dipole which is a bound-state of D0's and F1's to a D6-dipole which is a bound-state of D4's and F1's [23]. All the possible dipole domain walls will have a similar Gödel solution inside. A quick way to understand this is to look at the Gödel solution with the choice of RR-fields in eq. (2.4). If we lift this solution to M-theory, there is $U(3)$ symmetry which mixes the three two-planes x^6x^7, x^8x^9 and x^5x^{11}. The discrete subgroup of this symmetry which exchanges the two-planes becomes in our main solutions the U-duality which exchanges D0, D4 and F1 charges; this maps Gödel back to itself modulo some possible signs in the various forms.

Finally, there is the possibility of a domain wall which is the maximally spinning bound state of three charges. We call this microstate the hypertube. Presumably this state should correspond to a D0/D4/F1 bound-state discussed before. If we pick $n_0 \gg n_4 n_s$, we know that the state which saturates the angular momentum bound has $j^2 = 4n_0n_sn_4$. Unfortunately, in the right-moving sector this state will always carry some angular momentum in $U(1)_R$ and in the $x^{6..9}$ four-plane, and thus does not represent a perfect candidate for a domain wall which preserves an exact $U(1)_L \times SU(2)_R$ symmetry. We can, though, take two of these hypertubes and then pick a quantum state with all the angular momentum from the right-moving sector set to zero. We can also just smear a single hypertube.

The details of a hypertube domain-wall as realized on a D-brane are not yet known, and beyond the scope of this paper, but by clever inference we can draw some conclusions as to the properties of a collection of any number, N_H, of hypertubes. First of all, their angular

momentum is bounded by:

$$j \leq 2N_H \sqrt{\frac{n_0}{N_H}\frac{n_s}{N_H}\frac{n_4}{N_H}} = 2\sqrt{\frac{n_0 n_s n_4}{N_H}}. \tag{4.21}$$

which using our definitions implies

$$R \leq \sqrt{\gamma_0\gamma_s\gamma_4/N_H}\,\frac{1}{2\beta} \qquad \text{and} \qquad J^2 \leq 4Q_0Q_sQ_4/N_H. \tag{4.22}$$

A domain wall with N_H hypertubes always cuts off an under-rotating black hole, consistent with our understanding that with a little extra energy the N_H parts of the domain-wall could form a single bound state which would retreat behind the horizon of this putative black hole. The single hypertube exactly saturates the rotation bound, where the black hole horizon area disappears, leaving a diminished number of possible microstates. We also see that hypertube domain walls outside the horizon always bound a Gödel universe with radius $R \leq 1/2\beta$.

5 The Decoupling Limit

In this section, we will look at how our domain wall matching plays out in the near-horizon limit of the BMPV black holes we consider. For the two-charge solution, there is really no horizon, since the dilaton blows up as we take $\tilde{r} \to 0$. On the other hand, the U-dual D1/D5 system is a solution with a nice decoupling limit [22]. It is instructive to see what this U-dual limit does in our case. For the three-charge BMPV black hole, the limit is simpler, and yields a spacetime which is a deformation of $AdS_2 \times S^3 \times T^5$.

5.1 The U-duality Map and Decoupling Limit of the two-Charge Solution

The two-charge solution we wrote down in eq. (2.11) has a singularity at the origin, cut-off by our domain wall of course. One might understandably be shy of performing a "near-singularity" limit. Still, we will argue that we can take such limit smoothly by taking the $\gamma_i \to 1$. To help motivate this choice, we look at our solution through the looking-glass of the U-dual D1/D5 system, and infer the parameter and coordinate scalings of our solution from those of the Maldacena limit of its U-dual.

A chain of dualities which maps the the D0/F1 wrapping the $y(= x_9)$ direction system to the D5/D1 wrapping $\bar{x}_5, \ldots \bar{x}_9$ and $\bar{x}_5$ is as follows. First we T-dualize along the 678-directions to get D3/F1, then S-dualize to get D3/D1 and finally a T-duality along the 59-directions gets us to D5/D1. We denote the original T^4 radii $\tilde{L}$. The D1/D5 compact

directions have $\bar{R}_5 = T, \bar{R}_9 = S$ and $\bar{R}_{6\ldots 8} = K$. All these dualities are applied in the string frame, so the S-duality will introduce a non-trivial scale transformation involving the ratio:

$$\rho = \sqrt{\frac{ST}{\bar{g}_s l_s^2}} = \sqrt{\frac{\tilde{g}_s l_s^3}{\tilde{L}^3}}. \tag{5.1}$$

Using this ratio, we can now write the parameters and coordinates of the D1/D5 solution in terms of those of our D0/F1 charged black hole

$$S = \rho \frac{l_s^2}{\tilde{R}_9}, \quad T = \rho \frac{l_s^2}{\tilde{R}_5}, \quad K = \rho^{-1} \frac{l_s^2}{\tilde{L}}, \quad \bar{g}_s = \frac{l_s^2}{\tilde{R}_5 \tilde{R}_9}, \tag{5.2}$$

and

$$\bar{x}_{0\ldots 4} = \rho^{-1}\, \tilde{x}_{0\ldots 4}, \qquad \bar{Q}_i = \rho^{-2} Q_i. \tag{5.3}$$

We will now show that the Maldacena limit corresponds after U-duality to a scaling of our solution which takes $\gamma_s, \gamma_0 \to 1$ while keeping fixed the Gödel geometry and couplings in the near-tube coordinates. We call this the decoupling limit.

The Maldacena limit scaling can be written as taking $\varepsilon \to 0$ with

$$\bar{r} \to \varepsilon \bar{r}, \quad \bar{t} \to \varepsilon^{-1} \bar{t}, \quad \bar{T} \to \varepsilon^{-1} \bar{T}, \quad \bar{K}, \bar{S} \to \varepsilon^0 \bar{K}, \bar{S} \tag{5.4}$$

We can model this scaling for our coordinates as follows. The first step is to write both γ_0 and γ_s as $1 - \varepsilon^2$, then we write the asymptotic coordinates in terms of the near-tube ones as:

$$\begin{aligned} &Q_i = \varepsilon^{-1} R^2, \ \tilde{r} = \varepsilon^{1/2} r, \quad \tilde{t} = \varepsilon^{-3/2}\, t, \quad \rho = \varepsilon^{-1/2} \sqrt{g_s l_s^3 / L^3} \\ &\tilde{L} = \varepsilon^{1/2} L, \quad \tilde{R}_5 = \varepsilon^{1/2} R_5, \quad \tilde{R}_9 = \varepsilon^{-1/2} R_9, \end{aligned} \tag{5.5}$$

Plugging these values into Eqs. (5.2) and (5.3), we see that we get

$$\begin{aligned} &\bar{Q}_i = \frac{L^3}{g_s l_s^3} R^2, \quad \bar{r} = \varepsilon \sqrt{\frac{L^3}{g_s l_s^3}} r, \quad \bar{t} = \varepsilon^{-1} \sqrt{\frac{L^3}{g_s l_s^3}} t, \\ &\bar{T} = \varepsilon^{-1} \sqrt{\frac{g_s l_s^7}{R_5^2 L^3}}, \quad \bar{S} = \sqrt{\frac{g_s l_s^7}{R_9^2 L^3}}, \quad \bar{K} = \sqrt{\frac{g_s l_s^7}{L^5}}, \end{aligned} \tag{5.6}$$

just as required to satisfy Eq. (5.4).

After we take the $\gamma_i \to 1$ limit, or near-horizon limit, we get new metric and potentials which (after some gauge transformations) look like

$$\begin{aligned} ds^2 &= -\frac{r^3}{R^3} (dt + \frac{R^3}{4r^2}\, \sigma_3)^2 + \frac{r}{R}\, dy^2 + \frac{R}{r} \sum_{i=1}^{8} (dx^i)^2, \qquad (5.7) \\ B_2 &= -\frac{r^2}{R^2} (dt + \frac{R^3}{4r^2}\, \sigma_3) \wedge dy + dt \wedge dy, \qquad e^\Phi = g_s \sqrt{\frac{R}{r}}, \\ C_1 &= -\frac{r^2}{R^2} (dt + \frac{R^3}{4r^2}\, \sigma_3) + dt, \qquad C_3 = \frac{R}{4}\, dt \wedge dy \wedge \sigma_3. \end{aligned}$$

This solution still has a singularity at $r = 0$, but it no longer has any closed time-like curves. The singularity will be removed by our domain wall at $r = R$. From eq. (4.9), we see that

$$R = 8N_2 G_6 \tau_2^{-1} \tag{5.8}$$

and so can be made arbitrarily large by increasing the number of D2-branes.

In the U-dual D1/D5 background, the asymptotic geometry is locally $AdS_3 \times S^3 \times T^4$, and our D2-brane shell now becomes a KK-monopole [34]. Since the KK-monopole is a supergravity object, when $N_2 = 1$ that background is actually smooth, and the domain wall disappears.

5.2 The Three-Charge Near Horizon Limit

Now that we have identified the right procedure for the decoupling limit in the two-charge solution, it is simple to extend this procedure to the three charge black hole. We just switch to the near-tube coordinates and then take the limit $\gamma_i \to 1$. This can be checked using the U-duality above, or more simply, by T-dualizing along the y direction. For our choice of domain walls, this limit gives:

$$U, V, W \to \frac{R^2}{r^2}, \qquad \beta R \to \frac{1}{2}. \tag{5.9}$$

Then the metric and fields (modulo gauge transformations) for the outside solution takes the simple form

$$\begin{aligned}
ds^2 &= -\frac{r^4}{R^4}(dt + \frac{R^3}{4r^2}\,\sigma_3)^2 + dy^2 + R^2\frac{dr^2}{r^2} + \frac{R^2}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + \sum_{i=5}^{8}(d\tilde{x}^i)^2, \qquad (5.10)\\
B_2 &= -\frac{r^2}{R^2}(dt + \frac{R^3}{4r^2}\,\sigma_3) \wedge dy + dt \wedge dy, \qquad e^\Phi = g_s,\\
C_1 &= -\frac{r^2}{R^2}(dt + \frac{R^3}{4r^2}\,\sigma_3) + dt, \quad C_3 = \frac{R^2}{4}\cos\theta\, d\phi \wedge d\psi \wedge dy + \frac{R}{4}\, dt \wedge dy \wedge \sigma_3
\end{aligned}$$

which is a deformation of $AdS_2 \times S^3 \times T^5$. We expect this space-time to be dual to some deformation of a superconformal quantum mechanics. It would be interesting to understand the appearance of a domain wall at $r = R$ in the supergravity solution in terms of that quantum mechanics. Alternatively, a T-duality takes this background to a geometry which is locally $AdS_3 \times S^3 \times T^4$. The appearance of our domain wall now corresponds to the nucleation of long D-strings very similar to those in [36].

6 Supertube Quantum Mechanics, Hypertubes and Gödel Holography

In order to better understand how our domain wall solutions relate to micro-states of BMPV black holes and AdS geometries, a careful examination of these domain walls as states in the D2-brane DBI theory is in order. The explicit states we have seen so far are smeared D2-branes; we will contemplate this smearing in more detail.

A circular supertube is a compact extended object. As such, it has not just a center-of-mass location, but also a size, orientation, and a world-volume gauge field. It's world-volume fields include four periodic scalars whose vevs determine its location on T^4; we smear by averaging over this location. In R^4, the supertube is a circular ring specified by a position vector, a radius R and an orientation which takes values in S^2. All of these are scalars of the D2-brane worldvolume theory. The position decouples from the dynamics which interest us, therefore we will ignore it. As demonstrated in [5, 6], the radius field R has a non-zero vev due to a potential which depends on the charges Π and B. The circle which the supertube wraps is non-trivially fibered over the orientation S^2: this is actually the Hopf fibration with total space S^3. This implies that angular momentum in one two-plane is parallel transported into other two-planes as the tube moves in S^2. Smearing our supertube source over the orientation S^2 then gives it angular momentum in $U(1)_L$ and preserves the $SU(2)_R$ symmetry of our supergravity solutions.

A single smeared supertube can carry $SO(4)$ angular momentum with rank 4, while a localized supertube is restricted to rank 2 angular momenta. In order to accomplish our smearing over S^2 we would have to use some quantum mechanical averaging whose exact nature is not immediately clear[5]. More conventionally, a collection of a large number of N_2 separate supertubes, distributed around the S^2 and smeared along the flat directions, can also give a source with rank 4 angular momentum [23]. When N_2 is sufficiently large, we can spread the supertubes out on T^4 and the orientation S^2, obtaining a good classical approximation of N_2 smeared supertubes. Since rings with different orientations in R^4 preserve the same supersymmetries, the rings in our collection feel no relative force and retain the same energy as N_2 smeared supertube. The advantage of using a collection of N_2 un-smeared supertubes is that we can trust the supergravity solution a lot closer to the domain wall than when we try to pile up N_2 individually smeared supertubes. In the latter case, we should not trust our solution any closer than the size of region we smear over, which grows like R. In the former case, we can trust our solution up to the spacing, ℓ, separating any

[5] The corresponding state may not even be a pure state

two supertubes, which is of order

$$\ell = \Big(\frac{\pi R^2 V}{N_2}\Big)^{1/6}. \tag{6.1}$$

For hypertubes, less smearing is required. Whereas a supertube with a dust of co-moving T^4-wrapped D4-branes has angular momenta j_L and j_R both of the same order, and thus tends to extend in only one two-plane, once the tube binds with the D4-branes to form a supersymmetric hypertube we can only have one large angular momentum, either j_L or j_R. For N_H hypertubes we have

$$j_L \leq 2\sqrt{n_0 n_s n_4}/N_H^{1/2}, \qquad j_R \leq N_H \tag{6.2}$$

This means that for $n_i \gg N_H$, hypertubes have mostly rank 4 angular momenta and will extend in all the R^4 directions. Hypertubes are necessarily domain walls! This complements well with the U-dual results of [37].

6.1 Limits of Validity: Theory on Supertube

Our construction provides a mechanism for resolving the problem of closed time-like curves in the Gödel universe and in over-rotating BMPV black holes. In fact, we see that the two classical solutions solve each other's causality problems. This involves the appearance of smeared supertube domain wall between causally consistent portions of the two solutions, and possibly even more general objects. A natural question is that of the limits of validity, apart from those due to smearing, for such supergravity domain walls. Domain walls in general present a very interesting case for the duality between D-brane world-volume theories and the supergravity backgrounds that they source. As we increase the closed string coupling for D-branes with codimension larger then one, the surrounding background starts to curve, and a throat region emerges. For domain walls, the full gravitational backreaction is taken care of by solving the Israel matching conditions: no throat region appears. We can no longer think about replacing the D-brane with a dual near-horizon background, the D-brane and the curved background coexist. Stringy corrections can become important, of course, when the energy density of the domain wall reaches the string scale.

Our domain walls are in a sense an interesting hybrid because they are compact along three worldvolume dimensions. Far away, they look like bound states of D-branes which (after wrapping/smearing in the internal dimensions) appear approximately pointlike in the directions transverse to the noncompact dimensions of the domain wall. Hence, the back-reaction on the geometry makes their asymptotic fields identical to that of a black hole (or a continuation thereof, to the regime in which the naive classical solution would exhibit a

naked singularity surrounded by naked closed timelike curves). Once we get close to the domain wall, the back-reaction is instead encoded in the jump of the extensive curvature, which for our solutions is of order $1/R$. As long as R is large in string units, we don't need any stringy corrections and the thin wall approximation should hold. In our case, we can always make R sufficiently large, and the smearing less egregious, by increasing N_2, the number of overlapping supertubes[6]. On the other hand, for $N_2 = 1$, we have in the near-tube limit

$$R/l_s \propto g_s(l_s^4/V) \tag{6.3}$$

which will always be small in the perturbative regime $g_s \ll 1$. If we try to make $V \ll l_s^4$, we are forced to T-dualize and the problem remains. At this point, the region near the origin is inherently stringy, and is best described in terms of the non-commutative quantum mechanics of n_0 D0-branes [38].

We can further corroborate our view of the limits of validity if we consider the validity of the open string expansion for our domain wall. Since there is a non-trivial background B-field and magnetic field on the worldvolume of our supertubes, it is best to consider the expansion in the variables of Seiberg and Witten [39, 40]. Using these variables, we see that open string coupling for a single localized supertube is:

$$g_s\left(\frac{\det(\mathcal{G}+2\pi\alpha'\mathcal{F})}{\det(\mathcal{G})}\right)^{1/2} = g_s\,\frac{B}{R}. \tag{6.4}$$

If we take into account the fact that we have have N_2 supertubes, as well as the fact that these are spread[7] over a volume $\pi R^2 V$, we see that this open string coupling needs to be adjusted by a multiplicative factor which takes the form (up to minor numerical factors):

$$N_2\,\frac{(2\pi\alpha')^3}{\pi R^2 V} = \pi^{-2} g_s^{-1}\beta l_s. \tag{6.5}$$

This means that the open string coupling constant for the smeared supertubes is approximately

$$G_o = \frac{\gamma_0}{2\pi^2 R} l_s, \tag{6.6}$$

which is of the same order as the jump in the extrinsic curvature of the domain wall in string units. Therefore, we see that our whole picture fits together quite well for small g_s and large R. For small N_2, we once again have to go to the quantum-mechanical description in terms of D0-branes.

[6] For supertubes, the large N_2 limit corresponds to the long string limit of the U-dual D1/D5 system

[7] When we smear or spread D-branes, the number of choices for an open string to end on is effectively the density of D-branes in string units.

6.2 Gödel Holography

Now that we have a better understanding of what kind of domain walls we should use to cut-off a Gödel solution, we could speculate on what this means for applications holography to these solutions. By finding our domain walls, we have accomplished the first step in our systematic program to understand the quantum states of a stringy Gödel universe. Note that our construction allows one to make sense of Gödel solutions which are of *arbitrarily large* size compared to the string scale, simply by dialing N_2. We have also identified a near-wall limit which should prove useful in the process of isolating the stringy quantum states of one Gödel region of radius $1/2\beta$. For large N_2 and small β, we can characterize the degrees of freedom of the domain wall in terms of a non-commutative 2+1 field theory, while for small N_2 and large β, the description should be in terms of a D0-brane matrix quantum mechanics instead. One should be able to obtain and study the partition function exact for any N by generalizing the boundary states of [41].

If our domain walls indeed represent natural holographic screens for the Gödel universe, one interesting fact about them is that they are located at radius $1/2\beta$, instead of the value $\sqrt{3}/2\beta$ expected from the "phenomenological" analysis of preferred holographic screens as predicted by the behavior of massless, point-like probes of the classical Gödel geometry [1]. As we remarked on above, stringy probes certainly see the Gödel geometry somewhat differently than point-like probes, and a possible finite renormalization of the location of the holographic screens should not be unexpected. Since our domain wall is made up of smeared objects wound along orbits with line elements proportional to σ_3, it perhaps not so surprising that our screens appear before the radius grows to the value predicted by the point-like probes.

7 Final Thoughts

We have argued in this paper that supertubes, and their more general cousins, provide a microscopic string-theory resolution of the causality problems found in a large class of supersymmetric classical solutions. In particular, we have demonstrated that the Gödel universe and the class of over-rotating BMPV black holes solve each other's causality problems, by joining along a supertube domain wall.

This mechanism has interesting implications both for the outside black-hole solution, and for the inside Gödel geometry. From the outside perspective, it provides a string theory resolution to a broad class of supersymmetric timelike naked singularities describing over-rotating black holes, which would be otherwise thrown away in classical general relativity by

the assumption of cosmic censorship.

The implications for the Gödel solution are perhaps even more profound. The classical Gödel solution is homogeneous in spacetime, and thus unique for a fixed value of its vorticity β, but suffers from closed timelike curves and therefore its consistency is in question. Based on the resolution mechanism presented in this paper, we propose that in full string theory the unique (but inconsistent) classical solution is replaced by an entire moduli space of causally consistent domain-wall resolutions. The moduli are given by the location and geometry of the domain wall that connects the causal part of the Gödel to a causal outside geometry. None of the consistent stringy resolutions is fully homogeneous in spacetime; in particular, space translations (and some of the formal supersymmetry) have been broken spontaneously by the domain wall.

We would like to finish by remarking on two striking philosophical elements which have made themselves apparent during the work described in this paper. The first of these is the different role that D-brane domain walls play in the duality between open and closed strings, in contrast to the more studied case of higher codimension D-branes. D-brane domain walls seem to exactly straddle this duality: for any coupling we always have both the D-brane with open strings and a curved space with closed strings. It would be very interesting to gain a more generic understanding of this phenomenon which would encompass all D-brane domain walls, from enhançons to D8-branes.

Second, we would like to spotlight the fact that supertubes (and in some instances more general objects that we referred to as hypertubes) with large charges carry angular momentum much more effectively then their constituents. This in turn implies that the bound-state extended object supports a given amount of angular momentum in a spatial region that is much more compact than the region that would be effectively occupied by a collection of the individual unbounded constituents carrying the same angular momentum. Perhaps this behavior is generic in string theory, and should be expect of more general extended objects, even neutral black ones.

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